



# Strong-coupling effects in a plasma of confining gluons

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## Abstract

The plasma consisting of confining gluons resulting from the Gribov quantization of the SU(3) Yang-Mills theory is studied using non-equilibrium fluid dynamical framework. Exploiting the Bjorken symmetry and using linear response theory a general analytic expressions for the bulk,  $\zeta$ , and shear,  $\eta$ , viscosity coefficients are derived. It is found that the considered system exhibits a number of properties similar to the strongly-coupled theories, where the conformality is explicitly broken. In particular, it is shown that, in the large temperature limit,  $\zeta/\eta$  ratio, scales linearly with the difference  $1/3 - c_s^2$ , where  $c_s$  is the speed of sound. Results obtained from the analysis are in line with the interpretation of the quark-gluon plasma as an almost perfect fluid.

**Keywords:** Kinetic theory, quark-gluon plasma, transport coefficients, Gribov-Zwanziger quantization

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## 1. Introduction

The enormous amount of data collected in the ultra-relativistic heavy-ion collision experiments in the last decade strongly suggests that a new strongly-coupled state of nuclear matter, the so called Quark-Gluon Plasma (QGP), is produced. The QGP's space-time evolution turns out to be very well described within the effective framework of relativistic fluid dynamics, with the properties of the system encoded in the equation of state and transport coefficients. These properties should, in principle, follow from the underlying theory of Quantum Chromodynamics (QCD), using, for example, numerical QCD simulations on the lattice (lQCD) [1–3] or Hard-Thermal-Loop resummed perturbation theory [4–7]. However, while the lQCD results on thermodynamic variables are quite precise nowadays, the results concerning the transport coefficients, in particular bulk,  $\zeta$ , and shear,  $\eta$ , viscosities, are still plagued with uncertainties which are too large to make firm statements [8, 9]. In view of these arguments any hints about transport properties of the QGP are more than welcome and have to be drawn from other theoretical considerations.

In this proceedings contribution we shortly review our main results of Refs. [10, 11], where, for the first time, the in- and out-of-equilibrium properties of a plasma consisting of confining gluons were studied. The plasma constituents were described by the dispersion relation resulting from the Gribov-Zwanziger

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(GZ) quantization procedure [12, 13] of the SU(3) Yang-Mills (YM) theory. In this way we attempt to study implications of non-Abelian structure of the gluon system on the dynamic properties of the plasma. In particular we focus on the bulk and shear viscosity coefficients, which are crucial for application of the viscous fluid dynamical modelling.

## 2. The Gribov dispersion relation

The GZ approach is based on a meticulous gauge fixing procedure of the quantized YM theory introduced by Gribov in Ref. [12], which results in the introduction of a new scale, the so called Gribov parameter,  $\gamma_G$ , in the framework. The  $\gamma_G$  parameter controls the onset of the confinement effects in the gluon system, and it is found from the self-consistent solution of the gap-equation [12, 13]. As a result the dispersion relation of interacting gluons, which in the Coulomb gauge reads [12]

$$E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + \frac{\gamma_G^4}{\mathbf{k}^2}}, \quad (1)$$

improves the behavior of the theory in the infrared regime. The theory was generalized to finite temperatures by Zwanziger in Ref. [14], where its qualitative agreement with the pure glue lattice results was shown. Recently, the GZ approach received significant interest from the theory side resulting in several interesting findings of QCD under extreme conditions [15–22], which make this approach a promising playground for the study of confinement effects in YM realtime dynamics.

## 3. Bulk and shear viscosity of confining gluons

In order to apply our fluid dynamical considerations we first rewrite Eq. (1) in the Lorentz covariant form,  $E(k \cdot u) = \sqrt{(k \cdot u)^2 + \gamma_G^4/(k \cdot u)^2}$ , where  $\mathbf{k}$  is the three-momentum of a gluon,  $k^0 = |\mathbf{k}|$ , and  $u^\mu$  is the four-velocity of the rest frame. The latter formula may then be used in the finite temperature results for thermodynamic variables found by Zwanziger in Ref. [14]. For the calculation of transport coefficients, without loss of generality, we may impose the Bjorken symmetry [23] of the system, that is we consider simple longitudinally boost-invariant and transversally homogeneous (0+1)-dimensional expansion. In this case  $u^\mu = x^\mu/\tau$  and all thermodynamic variables become functions of proper time,  $\tau = \sqrt{t^2 - z^2}$ , solely.

Our approach is based on the relaxation time approximation (RTA) for the collision kernels [24]. Considering small perturbations around the equilibrium distribution function,  $f = f_{\text{GZ}} + \delta f$ , and making use of the first-order viscous fluid dynamical expressions for bulk and shear viscous pressure corrections, after straightforward algebra, we find the following expressions [10, 11]

$$\zeta = \frac{g_0 \gamma_G^5}{3\pi^2} \frac{\tau_{\text{rel}}}{T} \int_0^\infty dy \left[ c_s^2 - \frac{1}{3} \frac{y^4 - 1}{y^4 + 1} \right] f_{\text{GZ}}(1 + f_{\text{GZ}}), \quad (2)$$

$$\eta = \frac{g_0 \gamma_G^5}{30\pi^2} \frac{\tau_{\text{rel}}}{T} \int_0^\infty dy \frac{(y^4 - 1)^2}{y^4 + 1} f_{\text{GZ}}(1 + f_{\text{GZ}}), \quad (3)$$

for the bulk and shear viscosity, respectively. Above  $f_{\text{GZ}} = \{\exp[\gamma_G \sqrt{y^2 + y^{-2}}/T] - 1\}^{-1}$ ,  $c_s$  is the speed of sound,  $g_0$  is the degeneracy factor and  $\tau_{\text{rel}}$  is the relaxation time.

In the left panel of Fig. 1 we present the temperature dependence of  $\zeta$  (dashed-dotted line) and  $\eta$  (dotted line) calculated from Eqs. (2)–(3) and scaled by the equilibrium entropy density,  $s$ , and the relaxation time. Similarly to the ideal massless gas one observes approximately linear scaling of scaled shear viscosity,  $\eta/s \sim T\tau_{\text{rel}}$ , with temperature. Another interesting observation is the enhancement of scaled bulk viscosity around the critical temperature  $T_c = 260$  MeV, which highlights the importance of bulk viscosity for the heavy-ion phenomenology due to cavitation in the plasma [25]. Finally, unlike in the quasiparticle models, at low temperatures the bulk and shear viscosity become comparable.

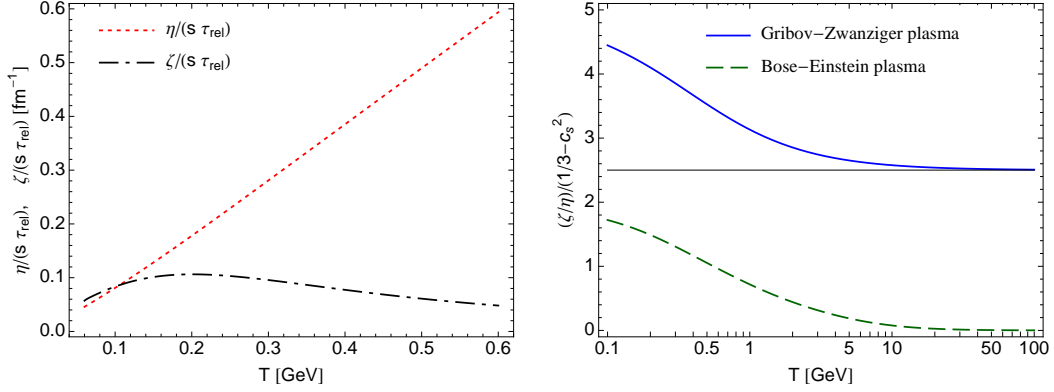


Fig. 1: Left panel: The temperature dependence of bulk (black dashed-dotted line) and shear (red dotted line) viscosity obtained within the linear response theory, Eqs. (2)–(3), scaled by the equilibrium entropy density and the relaxation time. Right panel: The  $\zeta/\eta/(1/3 - c_s^2)$  scaling as a function of temperature for the Gribov-Zwanziger plasma (solid blue line) and the massive Bose-Einstein plasma (dashed green line).

In the right panel of Fig. 1 we plot the ratio  $\zeta/\eta$  scaled by the factor  $1/3 - c_s^2$  vs. temperature for the GZ plasma (solid line) and a massive non-interacting Bose-Einstein (BE) plasma (dashed line). We note that the ratio  $\zeta/\eta$  is independent of the relaxation time. We observe that the values predicted within GZ formalism are always larger than the ones obtained for BE plasma. Moreover, we see that the scaling is approximately linear at asymptotically large temperatures.

The latter can be shown analytically by performing large- $T$  expansions of the shear and bulk viscosities. In this way for  $T \gg \gamma_G$  we arrive at the following scaling

$$\frac{\zeta}{\eta} = \kappa_{\text{GZ}} \left( \frac{1}{3} - c_s^2 \right), \quad (4)$$

where  $\kappa_{\text{GZ}} = 5/2$ . One should note here that the scaling (5) is characteristic for strongly-coupled theories based on gauge-gravity duality [26, 27], and disagrees with weakly-coupled approaches [28]. Equation (5) is qualitatively different from the scaling found for the BE plasma where

$$\frac{\zeta}{\eta} = \kappa_{\text{BE}} \left( \frac{1}{3} - c_s^2 \right)^{3/2} \quad (5)$$

and  $\kappa_{\text{BE}} = 3\sqrt{15}/2$ . On the other hand, at low temperatures we get  $\zeta/\eta = 5/3$  and  $\zeta/\eta = 2/3$  for GZ and BE plasma, respectively.

#### 4. Conclusions

In this proceedings contribution we have reviewed our main results on the non-equilibrium properties of the Gribov-Zwanziger plasma of confining gluons [12, 13] found in Refs. [10, 11]. In particular, we have emphasized certain features of the bulk and shear viscosities, such as the temperature dependence of their ratio, which are characteristics of the strongly-coupled theories [26, 27]. These results were confronted with those obtained for the non-interacting massive Bose-Einstein plasma.

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